AP Statistics Tutorial: Confidence Intervals

Statisticians use a confidence interval to describe the amount of uncertainty associated with a sample estimate of a population parameter.

How to Interpret Confidence Intervals

Consider the following confidence interval: We are 90% confident that the population mean is greater than 100 and less than 200.

Some people think this means there is a 90% chance that the population mean falls between 100 and 200. This is incorrect. Like any population parameter, the population mean is a constant, not a random variable. It does not change. The probability that a constant falls within any given range is always 0.00 or 1.00.

The confidence level describes the uncertainty associated with a sampling method. Suppose we used the same sampling method to select different samples and to compute a different interval estimate for each sample. Some interval estimates would include the true population parameter and some would not. A 90% confidence level means that we would expect 90% of the interval estimates to include the population parameter; A 95% confidence level means that 95% of the intervals would include the parameter; and so on.

Confidence Interval Data Requirements

To express a confidence interval, you need three pieces of information.

- Confidence level
- Statistic
- Margin of error

Given these inputs, the range of the confidence interval is defined by the sample statistic + margin of error. And the uncertainty associated with the confidence interval is specified by the confidence level.

Often, the margin of error is not given; you must calculate it. Previously, we described how to compute the margin of error.

How to Construct a Confidence Interval

There are four steps to constructing a confidence interval.

1. Identify a sample statistic. Choose the statistic (e.g., mean, standard deviation) that you will use to estimate a population parameter.

2. Select a confidence level. As we noted in the previous section, the confidence level describes the uncertainty of a sampling method. Often, researchers choose 90%, 95%, or 99% confidence levels; but any percentage can be used.

3. Find the margin of error. If you are working on a homework problem or a test question, the margin of error may be given. Often, however, you will need to compute the margin of error, based on one of the following equations.

   \[
   \text{Margin of error} = \text{Critical value} \times \text{Standard deviation of statistic}
   \]

   \[
   \text{Margin of error} = \text{Critical value} \times \text{Standard error of statistic}
   \]

   For guidance, see how to compute the margin of error.
Specify the confidence interval. The uncertainty is denoted by the confidence level. And the range of the confidence interval is defined by the following equation.

Confidence interval = sample statistic ± Margin of error

The sample problem in the next section applies the above four steps to construct a 95% confidence interval for a mean score. The next few lessons discuss this topic in greater detail.

### Sample Planning Wizard

As you may have guessed, the four steps required to specify a confidence interval can involve many time-consuming computations. Stat Trek's Sample Planning Wizard does this work for you quickly, easily, and error-free. In addition to constructing a confidence interval, the Wizard creates a summary report that lists key findings and documents analytical techniques.

Whenever you need to construct a confidence interval, consider using the Sample Planning Wizard. The Sample Planning Wizard is a premium tool available only to registered users.

Learn more

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**Test Your Understanding of This Lesson**

**Problem 1**

Suppose we want to estimate the average weight of an adult male in Dekalb County, Georgia. We draw a random sample of 1,000 men from a population of 1,000,000 men and weigh them. We find that the average man in our sample weighs 180 pounds, and the standard deviation of the sample is 30 pounds. What is the 95% confidence interval.

(A) 180 ± 1.86
(B) 180 ± 3.0
(C) 180 ± 5.88
(D) 180 ± 30
(E) None of the above

**Solution**

The correct answer is (A). To specify the confidence interval, we work through the four steps below.

- Identify a sample statistic. Since we are trying to estimate the mean weight in the population, we choose the mean weight in our sample (180) as the sample statistic.

- Select a confidence level. In this case, the confidence level is defined for us in the problem. We are working with a 95% confidence level.

- Find the margin of error. Previously, we described how to compute the margin of error. The key steps are shown below.

  - Find standard error. The standard error (SE) of the mean is:
    
    \[ SE = \frac{s}{\sqrt{n}} = \frac{30}{\sqrt{1000}} = 30/31.62 = 0.95 \]

  - Find critical value. The critical value is a factor used to compute the margin of error. To express the critical value as a *t* score (*t*), follow these steps.

    - Compute alpha (α): \( \alpha = 1 - \text{(confidence level / 100)} = 0.05 \)
    - Find the critical probability (p*): \( p^* = 1 - \alpha/2 = 1 - 0.05/2 = 0.975 \)
    - Find the degrees of freedom (df): \( df = n - 1 = 1000 - 1 = 999 \)
    - The critical value is the *t* score having 999 degrees of freedom and a cumulative probability equal to 0.975. From the *t* Distribution Calculator, we find that the critical value is 1.96.

    Note: We might also have expressed the critical value as a *z* score. Because the...
sample size is large, a z score analysis produces the same result - a critical value equal to 1.96.

- Compute margin of error (ME): \( ME = \text{critical value} \times \text{standard error} = 1.96 \times 0.95 = 1.86 \)

- Specify the confidence interval. The range of the confidence interval is defined by the \textit{sample statistic} \( + \text{margin of error} \). And the uncertainty is denoted by the confidence level. Therefore, we can be 95% confident that the population mean falls within the interval \( 180 + 1.86 \).